

A Comparison of NOOP to Structural Domain-Theoretic Models of OOP

Moez A. AbdelGawad
moez@cs.rice.edu

College of Mathematics and Econometrics, Hunan University
Changsha 410082, Hunan, P.R. China
Informatics Research Institute, SRTA-City
New Borg ElArab, Alexandria, Egypt

Abstract. Mainstream object-oriented programming languages such as Java, C#, C++ and Scala are all almost entirely nominally-typed. **NOOP** is a recently developed domain-theoretic model of OOP that was designed to include full nominal information found in nominally-typed OOP. This paper compares **NOOP** to the most widely known domain-theoretic models of OOP, namely, the models developed by Cardelli and Cook, which were structurally-typed models. Leveraging the development of **NOOP**, the comparison presented in this paper provides a clear and precise mathematical account for the relation between nominal and structural OO type systems.

1 Introduction

The first mathematical models of object-oriented programming (OOP) to gain wide-spread recognition were structural models. Being structural, objects were viewed in these models as being mere records. Object types, in accordance, were viewed as record types, where the type of an object specifies the structure of the object, meaning that object types carry information on the names of the members of objects (*i.e.*, fields and methods), and, inductively, on the (structural) types of these members. The model of OOP developed by Cardelli in the eighties of last century, and later enhanced by Cook and others, is an example of a structural model of OOP. Examples of structurally-typed OO languages include lesser-known languages such as O'Caml [41], Modula-3 [24], Moby [34], PolyTOIL [17], and Strongtalk [15].

Despite the popularity of the structural view of OOP among programming languages researchers, many industrial-strength mainstream OO programming languages are nominally-typed. Examples of nominally-typed OO languages include well-known languages such as Java [38], C# [3], C++ [2], and Scala [50]. In nominally-typed OO languages, objects and their types are nominal, meaning that objects and their types carry *class names information* (also called nominal information) as part of the meaning of objects and of their types, respectively.

In pure structurally-typed OO languages, nominal information is not used as part of the identity of objects and of their types during static type checking nor

is nominal information available at runtime¹. Accordingly, nominal information is missing in all structurally-typed models of OOP.

OOP was in its early days at the time the first mathematical models of OOP were developed, in the eighties of last century. Functional programming was the dominant programming paradigm among programming languages researchers at that time—and largely still is today. As such, the role of nominality of objects and of their types (*i.e.*, the inclusion of nominal information in their identities) in the semantics of mainstream OOP was not widely appreciated, and nominal OO type systems remain under-researched. **NOOP** [6,8] is a recently developed domain-theoretic model of OOP that addresses this shortcoming. To the best of our knowledge, **NOOP** is so far the only domain-theoretic model of OOP to include *full* class names information as found in mainstream nominally-typed OO programming languages. In this paper, we compare **NOOP** to other well-known structural domain-theoretic models of OOP.

This paper is structured as follows. First, we discuss related work—including the history of modeling OOP—in Section 2. As an appetizer for the following comparison, a fundamental technical difference between pure nominally-typed OO languages and pure structurally-typed OO languages is discussed in Section 3. In Section 4 we then compare the nominal mathematical view of OOP to the structural mathematical view of OOP by presenting a comparison between **NOOP** and the structural models of OOP constructed by Cardelli and enhanced by Cook and others. We conclude in Section 5 by summarizing our findings, making some final remarks, and discussing some possible future research.

2 Related Work

Even though object-oriented programming emerged in the 1960s, and got mature and well-established in mainstream software development in the late 1980s, the differences between nominally-typed and structurally-typed OO programming languages started getting discussed by programming languages (PL) researchers only in the 1990s [44,54,59]. In spite of these early research efforts, the value of nominal typing and nominal subtyping to mainstream OO developers did not get the full attention of the PL research community until around the turn of the century.

In the eighties, while OOP was in its infancy, Cardelli built the first denotational model of OOP [21,22]. Cardelli’s work was pioneering, and naturally, given the research on modeling functional programming extant at that time, the model Cardelli constructed was a structural denotational model of OOP.² In the

¹ Given that most industrial-strength OO languages are statically-typed, in this work we focus on nominal and structural *statically*-typed OO languages. A discussion of statically-typed versus dynamically-typed OO languages (including the non-well-defined so-called “duck-typing”), and the merits and demerits of each, is beyond the scope of this work. The interested reader should check [47].

² Quite significantly, Cardelli in fact also hinted at looking for investigating nominal typing [23, p.2]. Sadly, Cardelli’s hint went largely ignored for years, and structural

late 1980s/early 1990s, Cook and his colleagues worked to improve on Cardelli’s model, leading them to break the identification of the notions of inheritance and subtyping [28,31,30]. Unlike Cardelli, Cook emphasized in his work—as we discuss in more detail in Section 4.2—the importance of *self-references* in OOP, at the value level (*i.e.*, self variables, such as `this`) and at the type level (*i.e.*, self-type variables).

In 1994, Bruce et al. presented a discussion of the problem of binary methods in OOP [18]. Later, Bruce, and Simons, also promoted the structural view of OOP in a number of publications (*e.g.*, [19] and [58]) and they promoted conclusions based on this view. However, the deep disagreement between these conclusions (such as breaking the correspondence between inheritance and subtyping) and the fundamental intuitions of a significant portion of mainstream OO developers persisted [25,10].

Under the pressure of this disagreement, some PL researchers then started in the late 1990s/early 2000s stressing the significance of the differences between nominally-typed OOP and structurally-typed OOP, and they started acknowledging the practical value of nominal typing and nominal subtyping (see [10,7] for more details) and asserted the need for more research on studying nominal OO type systems [52]. Accordingly, some attempts were made to develop OO languages that are both nominally- and structurally-typed [33,51,37,45,46,50].³ However, at least in the eyes of mainstream OO developers, these hybrid languages have far more complex type systems than those of OO languages that are either purely nominally-typed or purely structurally-typed (see discussion in Section 4.1).

As to *operational* mathematical models of OOP, Abadi and Cardelli were the first to present such a model [4,5]. Their model also had a structural view of OOP. However, operational models of nominally-typed OOP got later developed. In their seminal work, Igarashi, Pierce, and Wadler presented Featherweight Java (FJ) [40] as an operational model of a nominally-typed OO language. Even though FJ is not the first operational model of nominally-typed OOP (for example, see [32], [49] and [35,36]), yet FJ is the most widely known operational model of (a tiny core subset of) a nominally-typed mainstream OO language, namely Java. The development of FJ and other operational models of nominally-typed OOP marked a strong focus on studying nominal-typing in OO languages, thereby departing from earlier disregard of it.

These developments later motivated the construction of **NOOP**. Featherweight Java (FJ) in fact offers the closest research to **NOOP** since it offers a very clear operational semantics for a tiny nominally-typed OO language. It is worth mentioning that **NOOP**, as a more foundational domain-theoretic model of nominally-typed OO languages (*i.e.*, that has fewer assumptions than FJ), provides a denotational justification for the inclusion of nominal information in

typing was rather *assumed* superior to nominal typing instead, particularly after the publication of Cook et al.’s and Bruce et al.’s work.

³ Multiple dispatch (see [26,14,27]), also, was discussed (*e.g.*, in [18]) as a possible solution to the problem of binary methods.

FJ. The inclusion of nominal information in **NOOP** is crucial for proving the identification of inheritance and subtyping in nominally-typed OOP. In FJ [40], rather than being proven as a consequence of nominality, the identification of inheritance and subtyping was taken as an assumption. **NOOP** also allows discussing issues of OOP such as type names, ‘self-types’ and binary methods on a more foundational level than provided by operational models of OOP. The more abstract description of denotational models results in a conceptually clearer understanding of the programming notions described, as well as of the relations between them.⁴

Finally, related to our work is also the dissatisfaction some researchers expressed about possible misunderstandings extant in the PL research community, and about the (mal)practices based on these misunderstandings when PL researchers study object-oriented programming languages in particular. Given the different basis for deriving data structuring in functional programming (based on standard branches of mathematics) and in object-oriented programming (based on biology and taxonomy) [21,22], some PL researchers have expressed dissatisfaction with assuming that the views of programming based on researching functional programming (including a view that assumes structural typing) may apply without qualifications to object-oriented programming. In addition to pointing out the importance of distinguishing between nominal typing and structural typing, MacQueen [42], for example, has noted many mismatches between Standard ML [48] (a popular functional programming language) and class-based OO languages such as Java and C++. Later, Cook [29] also pointed out differences between objects of OOP and abstract data types (ADTs), which are commonly used in functional programming.^{5,6}

⁴ It is worthy to also mention that **NOOP** was developed, partially, in response to the technical challenge Pierce (an author of FJ) presented in his LICS’03 lecture [53] in which Pierce looked for precisising the relation between structural and nominal OO type systems (notably, *after* the development of FJ was concluded).

⁵ We consider these research results as running in a similar vein as ours, since they somewhat also point to some mismatches between the theory and practice of programming languages—theory being mathematics-based, functional, and structurally-typed, and practice being biology/taxonomy-based, object-oriented, and nominally-typed.

⁶ Yet another research that is also somewhat similar to the one we present here, but that had different research interests and goals, is that of Reus and Streicher [55,57,56]. In [56], an untyped denotational model of class-based OOP is developed. Type information is largely ignored in Reus and Streicher’s work (in particular, members of objects have no type signatures) and some minimal amount of nominal information is included with objects only to support analyzing OO dynamic dispatch. This model was developed to analyze mutation and imperative features of OO languages and for developing specifications of OO software and the verification of its properties [56]. Analyzing the differences between structurally-typed and nominally-typed OO type systems was *not* a goal of Reus and Streicher’s research. Despite the similarity of **NOOP** and the model of Reus and Streicher, we thus make no further mention of Reus and Streicher’s model in this paper due to its different interests and goals, and due to the fundamentally different nature of **NOOP** compared to their

3 Type Names, Type Contracts, Recursive Types and Binary Methods

From the point of view of OO developers and OO language designers, there are many technical differences between nominally-typed OO languages and structurally-typed OO languages. We discuss these in brief in this section. (A more detailed discussion is presented in [10] and [7].)

Type Names and Behavioral Type Contracts A fundamental technical difference between nominally-typed OO type systems and structurally-typed OO type systems is how the two approaches view type names. In structurally-typed OO languages, type names are viewed as being names for type variables that *abbreviate type expressions* (*i.e.*, are “shortcuts”). As such, the use of type names in structurally-typed OO languages is not always necessary, but type names are useful as abbreviations and they are even necessary for defining recursive type expressions. As variable names, however, recursive type names in structurally-typed OO languages (such as the name of a class when used inside the definition of the class—which gets interpreted as “self-type”) get *rebound* to different types upon type inheritance, and they get rebound to types that, if they were subtypes, could break the contravariant subtyping rule of method parameter types (and, thus, break the type safety of structurally-typed OO languages). Structurally-typed OO languages resolve this situation by breaking the correspondence between type inheritance and subtyping.

In nominally-typed OO languages, on the other hand, the nominality of types means type names are viewed as part of the identity and meaning of type expressions, since type names in these languages are associated with public formal or informal *behavioral contracts*.⁷ Being names for public, and thus *fixed*, contracts means that, in nominally-typed OO languages, type names cannot be treated as variable names. In nominally-typed OO languages, thus, type names have fixed meanings that do *not* change upon inheritance. Further, in these languages the fixed type a type name is bound to does *not* break the contravariant subtyping of method parameters when the method and its type get inherited by subtypes (types corresponding to subclasses/subinterfaces). As such, in nominally-typed

model (*i.e.*, **NOOP** including all essential class names information inside objects versus Reus and Streicher’s model lacking most of this information.)

⁷ In well-designed OO programs, each class (and interface and trait, in languages that support these notions) has associated contracts describing the behavior of objects of the class (its instances). The contracts include an invariant (a predicate) for the values of class fields, and a contract for each method stipulating what conditions the inputs should satisfy and what output condition should hold over the value returned by the method, as well as side effects that have been performed on this and perhaps other objects passed as arguments to the method. (The output predicate may mention the values of arguments and in such case is often called an input-output predicate.) In practice, class contracts are typically informal, may be incomplete, and are usually expressed only in code documentation. (See [10] for a longer, detailed and deeper discussion of the association of type names with contracts, and of the import of this association to mainstream OO developers.)

OOP it is not necessary to break the identification of type inheritance with subtyping.

Recursive Types Further, in class-based OOP, a class (or interface or trait, in languages that support these notions) can directly refer to itself (using class/interface/trait names) in the signature of a field, or the signature of a method parameter or return value, where the class name is used also as a type name. This kind of reference is called a type *self-reference*, *recursive reference*, or, sometimes, *circular reference*. Also, mutually-dependent classes, where a class refers to itself indirectly (*i.e.*, via other classes), are allowed in class-based OOP. As Pierce noted [52], nominally-typed OO languages allow readily expression of mutually-dependent class definitions. Since objects are characterized as being self-referential values (according to Cook [29], objects are ‘autognotic’), and since self-referential values can be typed using recursive types [43], there is wide need for recursive type definitions in mainstream OOP. As such, direct and indirect circular type references are quite common in mainstream OOP [29]. The ease by which recursive typing can be expressed in nominally-typed OO languages is one of the main advantages of nominally-typed OOP.⁸

In the comparison of nominal and structural mathematical models of OOP in Section 4 we will see that, in accordance with their different views of type names, self-referential class references are viewed differently by nominally-typed models of OOP than by structurally-typed models of OOP. The different views of circular class references are behind nominal models of OOP leading to a different conclusion about the relation between inheritance and subtyping than the conclusion reached based on structural models.

Binary Methods From the point of view of OO developers, the difference between the nominal and the structural views of type names in OOP demonstrates itself, most prominently, in the different support and the different treatment provided by OO languages to what are usually called “binary methods”. In OOP, a ‘binary method’ is defined as a method that takes a parameter (or more) of the same type as the class the method is declared in [18]. “The problem of binary methods” and requiring them to be supported in OO languages was a main motivation behind structural models of OOP leading to inheritance and subtyping not being identified (*i.e.*, as not being in a one-to-one correspondence) [30]. As explained above, given their view of type names as type variable names, structurally-typed OO languages require the self-type of the argument of a method—where the method is identified as a binary method, and upon inheritance of the method by a subclass of the class the method is first declared in—to be that of the type corresponding to the *subclass*.

Nominally-typed OO languages, on the other hand, with their fixed interpretation of type names, treat a method taking in an argument of the same class as that in which the method is declared like any other method, *i.e.*, needing no special treatment. As such, nominally-typed OO languages guarantee that the

⁸ According to Pierce [52, p.253], “The fact that recursive types come essentially for free in nominal systems is a decided benefit [of nominally-typed OO languages].”

type of the input parameter of a method that approximates a binary method is a *supertype* of its type if it were a true binary method.

Nominally-typed OO languages, thus, offer a somewhat middle-ground solution between totally avoiding binary methods and overly embracing them (as pure structurally-typed OO languages do). Given that the meaning of types names in nominally-typed OO languages does not change upon inheritance, these languages provide methods whose type, upon inheritance, only approximates the type of true binary methods. Nominally-typed OO languages do not quite support binary methods, but, for good reasons (*i.e.*, so as to not break the identification of inheritance of contracts and subtyping, nor lose other advantages of nominal typing [10]), offer only a good approximation to binary methods. Given that the type of the parameter does not change in subclasses, the degree of approximation (if the method was indeed a true binary method) gets lesser the deeper in the inheritance hierarchy the method gets inherited.⁹

4 Nominally-Typed versus Structurally-Typed Models of OOP

To see how nominality and nominal typing affects mathematical views of OOP, we compare **NOOP**, as a nominally-typed denotational model of OOP, to the most widely known structural model of OOP—the one constructed and presented by Cardelli [21,22], and extended, analyzed and promoted by others such as Cook [28,31,30], Bruce [19] and Simons [58].

Even though unnamed by their authors, for ease of reference in this paper we call Cardelli’s model **SOOP**, for Structural OOP, while calling the extension of **SOOP** by Cook et al. μ **SOOP** (due to its inclusion of recursive types). As we discussed earlier, **NOOP** is the first domain-theoretic model of OOP to include full nominal type information found in nominally-typed OOP. The construction of **NOOP** is presented in [6], and is summarized in [8]. In the following sections we first compare **NOOP** to **SOOP** then compare it to μ **SOOP**.

⁹ With the introduction of generics [38,3,50,13,11,16,40], and ‘F-bounded generics’ (the nominal counterpart of F-bounded polymorphism [20,12,39]) in particular, nominally-typed OO languages provided better support for *true* binary methods while keeping the identification of type inheritance with subtyping and other benefits of nominal typing. It should be noted that the lesser-recognized problem of ‘spurious binary methods’ in structurally-typed OOP (see [10, Section 3.3.1]) provides further justification for nominally-typed OO languages being cautious about fully embracing binary methods by treating a method that “looks like” a binary method as indeed being one. In light of the spurious binary methods problem, and precluding the use of F-bounded generics, in our opinion a better approach towards supporting true binary methods in mainstream OO languages might be by allowing developers to *explicitly* mark or flag true binary methods as being such, or, even more precisely, to allow developers to mark specific arguments of methods as being arguments that ‘need to be treated as those of true binary methods.’

4.1 NOOP Compared to SOOP

The model of OOP developed by Cardelli in the 1980s [21,22] was the first denotational model of OOP to gain widespread recognition. In his pioneering and seminal work Cardelli, according to him himself, had a goal of ‘unifying functional programming and object-oriented programming’ [22, p.2]. A domain equation that describes the main features of **SOOP** (distilled to exclude variants. See [22, pp.15, 16] for the actual domain equations used by Cardelli) is

$$\mathcal{V} = \mathcal{B} + (\mathcal{V} \rightarrow \mathcal{V}) + (\mathcal{L} \rightarrow \mathcal{V})$$

where \mathcal{V} is the main domain of values, \mathcal{B} is a domain of base values, \mathcal{L} is the flat domain of labels, \rightarrow is the standard continuous functions domain constructor, and $+$ is the disjoint summation domain constructor. The distilled **SOOP** domain equation expresses the view that values are either base values, unary functions over values, or *records* (“objects”) modeled as (infinite) functions from labels to values.

The domain equation describing **NOOP** is

$$\mathcal{O} = \mathcal{S} \times (\mathcal{L} \multimap \mathcal{O}) \times (\mathcal{L} \multimap (\mathcal{O}^* \multimap \mathcal{O}))$$

where the main domain defined by the equation, namely domain \mathcal{O} , is the domain of (raw) objects, \times is the strict product domain constructor, and \multimap is the records domain constructor (See [8] or [6, Chapter 6] for more details on the **NOOP** domain equation). The **NOOP** domain equation expresses the view that every object is a triple of: (1) a class signature closure (*i.e.*, a member of domain \mathcal{S}), (2) a fields record (*i.e.*, a member of $\mathcal{L} \multimap \mathcal{O}$), and (3) a methods record (*i.e.*, a member of $\mathcal{L} \multimap (\mathcal{O}^* \multimap \mathcal{O})$, where \multimap is the strict continuous functions domain constructor, and $*$ is the finite-sequences domain constructor).

Class signatures and other related constructs are syntactic constructs that capture *all* nominal (*i.e.*, class/interface/trait names) information found in objects of mainstream nominally-typed OO software [8,6]. Class signatures formalize the informal notion of ‘object interfaces’ [10,7,6]. Embedding class signature constructs in objects of **NOOP** makes them nominal objects. It should be noted that consistency conditions for signature constructs in **NOOP** [8, Section 4] [6, Section 5.1] do not preclude a signature from directly or indirectly referring to itself in the signature of a field or of a method parameter or method return value, so as to allow for self-referential types (see Section 3.)

A comparison of **NOOP** to **SOOP** reveals the following fundamental difference between the two models:

- **SOOP** is a structural model of OOP, that, as explained by its domain equation, does *not* include nominal information into its objects. As such, **SOOP** views objects as being *essentially records* (of functions) [22, p.3]. Due to the lack of nominal information, the definitions of types of objects and of subtyping, based on **SOOP**, are also structural definitions, *i.e.*, ones that can only respect object structures but that *cannot* respect the behavioral contracts maintained by objects that are associated with their type names.

- **NOOP** is a nominal model of OOP, that, via the \mathcal{S} component (for signatures) of its domain equation, *includes full nominal information* into its objects. As such, **NOOP** views objects as records (of fields and methods) *accompanied by nominal information* referencing the behavioral contracts maintained by the fields and methods of the objects. The definition of types of objects and of subtyping, based on **NOOP**, can thus be nominal ones, *i.e.*, ones which *can* respect behavioral contracts associated with type names in addition to respecting object structures.

In the comparison of **NOOP** to **SOOP** it should also be noted that the ‘Inheritance \Leftrightarrow Subtyping’ (‘inheritance is subtyping’) theorem of **NOOP** ([8, Section 5.3]), stating the identification of type inheritance with subtyping in nominally-typed OOP, is very similar to Cardelli’s ‘Semantic Subtyping’ theorem ([22, Section 11]). Cardelli did not model recursive types, and thus did not handle recursive type expressions (which are the structural counterpart of self-referential class signatures). As such, despite the model of Cardelli being a structural model of OOP, the omission of recursive types enabled Cardelli to easily identify an inaccurate “structural” notion of inheritance with a structural definition of subtyping and prove their one-to-one correspondence¹⁰.

Other tangential differences that are noted in the comparison between **NOOP** and **SOOP** include:

1. **SOOP** models records as infinite functions, with only an informal restriction on the functions that requires the functions to map a cofinite set of input labels—*i.e.*, all but a finite number of labels—to the value **wrong**. **NOOP**, on the other hand, models the record component of objects using the \multimap (‘rec’) domain constructor which constructs records as tagged *finite* functions. Domain constructor \multimap , even though having similarity to some other earlier-developed domain constructors, was particularly developed to let **NOOP** model mainstream OOP more accurately. Because of using \multimap , the **NOOP** domain of objects formally includes no “junk” (*i.e.*, unnecessary) infinite records as those found in the formal definition of **SOOP**.
2. Given its attempt to unify FP and OOP, **SOOP** allows functions as first-class values in its main domain of values. As such, **SOOP** is not a pure-OO model of OOP. **NOOP**, on the other hand, is a pure-OO model of OOP. Every value in the main domain of **NOOP** is an object. To model methods and records, **NOOP** uses functional domains, but they are used only as auxiliary domains.
3. Functions (used to model methods) in **SOOP** are unary functions that take exactly one argument—an element of domain \mathcal{V} . **SOOP** thus requires ‘currying’ to model multi-ary functions and methods. In **NOOP**, on the other

¹⁰ In his work, Cardelli, informally and somewhat implicitly, defined inheritance as structural subtyping between (record) type expressions. Demonstrating the strong influence of functional programming on Cardelli’s model, Cardelli even argued for expanding the definition of inheritance to include some notion of “inheritance” between function types (by which it seems Cardelli really meant subtyping, since Cardelli did not suggest any code sharing).

hand, sequences of objects are used as method arguments to model multi-ary methods more precisely (*i.e.*, without the need for currying, which is not commonly familiar to mainstream OOP developers as it is to FP developers, and thus, inline with the previous point, also without the need for functions/methods to be first-class values).

4. **SOOP** uses the same namespace for fields and methods of records, disallowing a field in a record to have the same name as a method in the record. **NOOP**, on the other hand, aims to mimic mainstream OO languages more closely, and thus it uses two records as separate components inside objects to give fields and methods separate namespaces. A field and a method in a **NOOP** object can thus have the same name without conflict (method overloading, however, where two methods inside an object can have the same name, is supported neither by **SOOP** nor by **NOOP**).¹¹

Nominal vs. Structural vs. Hybrid Typed OO Languages It is worthy to mention here that the fundamental ‘structural versus nominal’ difference between **SOOP** and **NOOP** has profound implications on comparing nominally-typed OO languages to structurally-typed OO languages, and to hybrid OO languages that try or claim to support both nominal and structural typing.

First, it is clear that supporting nominal typing in an OO language with a structural view of objects is impossible, since the nominal information stripped by the structural view of objects is irrecoverable from the structure of the objects. Second, due to the association of type names to behavioral contracts, it is clear nominal typing is closer to semantic/behavioral typing than structural typing is (More discussion of contracts and semantic typing is presented in [10]).

Thirdly, from the definition of **NOOP** it is clear also that, if needed, it is easy to define structural types on a domain of nominal objects. The definition of these types can be done in **NOOP** by ignoring nominal information, as is done in “hybrid” OO languages such as Scala, SmallTalk, Whiteoak and Unity. The definition of these structural types in this case is not the same as for an OO language based on a structural view of objects and modeled by **SOOP**, since objects of the defined structural types will still carry nominal information at run-time (ready to be used during software run-time, such as in type casting operations and `instanceof` type tests). Structural OO languages that support a structural view of objects are fundamentally different than nominal languages because objects in such languages, as modeled by **SOOP**, are plain records (and thus without any reference to behavioral class contracts), which is *not* true in OO languages that try to support both nominal and structural types.¹²

¹¹ To put research on structural OOP on a more rigorous footing, and as a step towards the construction of **NOOP**, we constructed **COOP**—[6, Ch. 4] and [9, Sec. 4]—as a simple structural domain-theoretic model of OOP that dealt with the first three of the four tangential differences between **NOOP** and **SOOP**.

¹² A further reason we do not believe hybrid languages, such as Scala [50], SmallTalk [1], Whiteoak [37] and Unity [45], indeed provide true or full support for structural typing is that these languages do not quite support *recursive* structural types (varying

4.2 NOOP Compared to μ SOOP

Cook built on Cardelli's work by first developing a model of untyped inheritance [28,31] and, with others, then built a model of typed inheritance [30]. In his work, Cook took self-referential classes, and thus recursive types, in consideration, but, following the footsteps of Cardelli, Cook kept a structural view of OO typing. Thus Cook et al. concluded that 'inheritance is not subtyping' [30].

Building on the work of Cook et al. and based on its conclusions, Bruce, in his book on the foundations of OO languages [19], and Simons, in a series of articles on the theory of classification [58], enforced in the PL research community the conclusion reached by Cook and his colleagues regarding breaking the relation between inheritance and subtyping (implying the superiority of a structural view of OOP in the process), even when the conclusion opposed and contradicted the intuition (and even the "conventional wisdom" [30, p.125]) of a large section of OO developers and OO language designers. To explain the discrepancy, it was then thought that mainstream OO languages are technically deficient or flawed because, according to Cook [30], these languages 'place restrictions on inheritance'.

Given that μ SOOP (*i.e.*, Cook et al's work) is based on that of Cardelli, the differences between **NOOP** and **SOOP** we discussed in Section 4.1 get inherited by a comparison between **NOOP** and μ SOOP.

The main technical similarity between **NOOP** and μ SOOP is that both models of OOP take self-referential classes, and thus recursive types, in consideration. This is also where the two models strongly disagree, since **NOOP** leads to a different conclusion about the relation between inheritance and subtyping than μ SOOP does. This different conclusion is due to the differences in the nominal view of objects versus the structural view of them and to the inclusion/exclusion of contracts in object typing and object subtyping, and accordingly due to the role of inheritance (and thus contracts) in deciding subtyping.

As such, in addition to the main difference with **SOOP**, comparing **NOOP** to μ SOOP highlights the following four differences, which we first mention then discuss afterwards in some detail.

1. **NOOP** and μ SOOP have different views of type names.
2. **NOOP** and μ SOOP have different definitions of type inheritance.
3. **NOOP** and μ SOOP are different as to the uniformity of their inheritance models at the object level and at the type level.
4. **NOOP** and μ SOOP are different as to the simplicity of the mental model they present to developers during the OO software design process.

between having reluctant/weak support to having no support for them at all). As discussed in Section 3, recursive types are essential for serious OO programming. As demonstrated by Cook's work (which we discuss in more detail in the next section), supporting recursive structural types (and thus fully supporting structural typing in these so-called hybrid languages) leads to undesirable consequences. The interested reader is again advised to see [10] for more details.

Views of type names As we discussed, in detail, in Section 3, a main difference between nominal typing and structural typing that is illustrated by comparing **NOOP** to μ **SOOP** is how type names are viewed in nominal versus structural OO type systems, them having fixed meanings in the first, while allowing their meanings to get rebound (upon inheritance) in the latter.

Definitions of inheritance It is worthy to note that the different conclusion reached by **NOOP** than that by μ **SOOP** on the relation between inheritance and subtyping is based, in particular, on how the two models differently define inheritance. Cook defines inheritance as ‘a mechanism for the definition of new program units by modifying existing ones in the presence of self-reference’ [28]. Cook also intentionally targets modeling the multiple levels of inheritance that take place in OOP *uniformly* (as we discuss below), having a single model of inheritance that models type-level inheritance and object-level inheritance. Applied to types, Cook’s definition of inheritance based on a structural view of types makes type inheritance ‘a mechanism for the definition of new record type expressions by modifying existing ones, in the presence of ‘self-type’’. On the other hand, for the purpose of modeling nominally-typed mainstream OOP with a nominal view of types (as in **NOOP**), Cook’s definition of type inheritance has to be changed to ‘a mechanism for the definition of new class signatures by adding member (*i.e.*, field and method) signatures to an explicitly-specified set of existing class signatures.’

In contrast to Cook’s structural definition of type inheritance, the nominal definition of type inheritance, first, disregards self-types as having relevance in the definition, in agreement with the intuitions of mainstream OO developers about the inheritance of class signatures (where it is implied that nominal typing, with its fixed bindings of type names, only presents an approximation to self-types). Secondly, also in agreement with intuitions of mainstream OO developers, the nominal definition of type inheritance stresses *explicitness* in specifying inheritance, making inheritance an intended relation that is based on behavioral contracts and structure, not an accidental relation based only on structure.

Uniformity of inheritance models Structurally-typed OOP, as modeled by μ **SOOP**, uniformly applies the *same* model of inheritance (*i.e.*, Cook’s model [28]) at the level of values (*i.e.*, objects) and at the level of types. Using the same model at both levels requires rebinding the self-variable, at the value level, and rebinding of the self-type-variable, at the type level, upon inheritance. Nominally-typed OOP, and thereby **NOOP**, on the other hand, uses *two* different models of inheritance, one at the level of values (*i.e.*, objects) and another at the level of types. The model of inheritance at the level of values used in nominally-typed OOP (the model of [28] applies well) allows for rebinding the self-variable upon inheritance. At the level of types, however, a different model where type names do not get rebound is used by nominally-typed OOP, since there is no exact notion of a self-type-variable in nominally-typed OO languages (but only an approximation to it, using a superclass name, is available, as we explain in Section 3).

As such, while the model of inheritance used in μ **SOOP** uniformly applies to object-level inheritance and type-level inheritance, we can see that the models of inheritance used in **NOOP** reflect the non-uniformity of inheritance models in mainstream nominally-typed OOP, where a different model (and thus a different definition of inheritance) is used at the object level than that at the type level.

Economy of OO software design conceptual model Agreeing with the intuitions and conventional wisdom of mainstream OOP software developers and OOP language designers, **NOOP** proves that ‘inheritance is subtyping’ [6,8,25], *i.e.*, that there is a one-to-one correspondence between OO type inheritance and OO subtyping, while μ **SOOP** breaks the correspondence and proves that ‘inheritance is not subtyping’ [30,19,58]. Splitting inheritance from subtyping, as μ **SOOP** necessitates, requires a structurally-typed OOP developer to keep *two* hierarchies in mind when developing his software, namely, the inheritance hierarchy and the subtyping hierarchy¹³.

This complexity, and the disregard of class contracts in deciding subtyping, creates significant problems from the perspective of OO program design (See [10]). Respecting semantic class contracts in subtyping (thereby maintaining the identification of inheritance with subtyping) allows nominally-typed OOP developers on the other hand to reason about their software more readily and to keep only *one* hierarchy in mind while developing their software, leading to a simpler more economic software design conceptual model.

Table 1 on the following page summarizes the similarities and differences between **NOOP**, **SOOP** and μ **SOOP**.

5 Concluding Remarks and Future Work

The identification of types with behavioral contracts, and of subtyping with the inheritance and possible narrowing of contracts, makes nominal typing and nominal subtyping in nominally-typed OOP closer to semantic typing and semantic subtyping. Based on noting that, in this paper we compared a nominally-typed domain-theoretic model of OOP to the most well-known structurally-typed models. Our comparison has shown that nominally-typed models and structurally-typed models of OOP lead to different views of fundamental notions of object-

¹³ Bruce, in an attempt to address this issue, suggested that OO languages replace subtyping with ‘match-bounded polymorphism’ (which is a simplification of F-bounded polymorphism [20,12]) then identify type inheritance with matching. Matching [18], upon which match-bounded polymorphism depends, however, uses subtyping in its definition. As such, match-bounded polymorphism is not truly a full replacement of subtyping, since developers still need to understand subtyping to be able to understand matching. Having a non-simple mental model of OOP, due to insisting on maintaining the split between subtyping and inheritance, creates significant conceptual problems when designing OO software. We speculate that this led Bruce’s suggested language extensions on matching to not gain traction or support in mainstream OO languages.

	SOOP (Cardelli; 1980s)	μ SOOP (Cook et al; 1990s)	NOOP (AbdelGawad; 2010s)
<i>Nominal Info.</i>	Structural model; Class names info. missing from objects	Structural model; Class names info. missing from objects	Nominal model; Full class names info. included in objects
<i>Object Types</i>	Structural (reflect only object structure)	Structural (reflect only object structure)	Nominal (reflect struc. and assoc. contracts)
<i>Recursive Types</i>	Excluded	Included	Included
<i>View of Type Names</i>	Shortcuts. Self-ref. not considered	Shortcuts. Self-ref. gets rebound	Associated with public contracts. No rebinding
<i>Inheritance Models</i>	Redefine inheritance as non-recursive structural subtyping	Same rebinding model at object level and type level	Object level: Re- binding. Type level: No rebinding
<i>Type Inheritance</i>	Structural	Structural	Nominal
<i>Conceptual Economy</i>	Inher. = Subty.	Inher. \neq Subty.	Inher. = Subty.

Table 1. SOOP vs. μ SOOP vs. NOOP

oriented programming, namely objects, type names, class types, subtyping and the relation between subtyping and inheritance.

In particular, our comparison highlights that in nominally-typed OOP

1. An object should not be mathematically viewed as merely a records of its members (*i.e.*, its fields and methods) but rather as **a record together with nominal information** that is associated with class contracts that the object maintains—this information being carried along with the record, behaviorally constraining its members,
2. A class type should not be viewed as a record type but rather as **a record type that additionally respects behavioral contracts** associated with nominal information embedded in elements of the type (*i.e.*, its objects), and
3. Inheritance is correctly identified with nominal subtyping, *i.e.*, that in pure nominally-typed OOP **inheritance is subtyping**.

We believe the development of **NOOP**, and the mathematical comparison presented in this paper, are significant steps in providing a full account of the relation between nominal and structural OO type systems.

Further, we hope that having a more accurate mathematical view of nominally-typed OO software presents programming languages researchers with better chances for progressing mainstream OO programming languages. For example, generics ([38,3,50]) add to the expressiveness of type systems of nominally-typed OO programming languages ([13,11,16,40]). As hinted to earlier, we believe that F-bounded generics offer better support for binary methods in nominally-typed OO languages while maintaining the benefits of nominal typing. Building a domain-theoretic model of generic nominally-typed OOP, akin to **NOOP**, and comparing it to domain-theoretic models of polymorphic structurally-typed OOP, can as such offer better chances for having a deeper understanding of

features of generic mainstream OO languages such as generic binary methods, variance annotations (such as Java wildcards), Java erasure, polymorphic methods, and generic type inference.

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References

1. *ANSI Smalltalk Standard*. 1998.
2. *ISO/IEC 14882:2011: Programming Language: C++*. 2011.
3. C# language specification, version 5.0. <http://msdn.microsoft.com/vcsharp>, 2015.
4. Martin Abadi and Luca Cardelli. A semantics of object types. In *Proc. LICS’94*, 1994.
5. Martin Abadi and Luca Cardelli. *A Theory of Objects*. Springer-Verlag, 1996.
6. Moez A. AbdelGawad. *NOOP: A Mathematical Model of Object-Oriented Programming*. PhD thesis, Rice University, 2012.
7. Moez A. AbdelGawad. An overview of nominal-typing versus structural-typing in object-oriented programming (with code examples). Technical report, [arXiv.org:1309.2348](http://arxiv.org/abs/1309.2348) [cs.PL], 2013.
8. Moez A. AbdelGawad. A domain-theoretic model of nominally-typed object-oriented programming. *Journal of Electronic Notes in Theoretical Computer Science (ENTCS)*, DOI: 10.1016/j.entcs.2014.01.002. Also presented at *The 6th International Symposium on Domain Theory and Its Applications (ISDT’13)*, 301:3–19, 2014.
9. Moez A. AbdelGawad. Domain theory for modeling oop: A summary. Technical report, [arXiv.org:1406.7497](http://arxiv.org/abs/1406.7497) [cs.PL], 2014.
10. Moez A. AbdelGawad. Why nominal-typing matters in OOP. In *Submitted for publication in Onward! Essays*, 2016.
11. Ole Agesen, Stephen N Freund, and John C Mitchell. Adding type parameterization to the Java language, 1997.
12. Paolo Baldan, Giorgio Ghelli, and Alessandra Raffaeta. Basic theory of f-bounded polymorphism. *Information and Computation*, 153(1):173–237, 1999.
13. Joseph A. Bank, Barbara Liskov, and Andrew C. Myers. Parameterized types and Java. Technical report, 1996.
14. John Boyland and Giuseppe Castagna. Parasitic methods: An implementation of multi-methods for Java. In *OOPSLA*, 1997.
15. G. Bracha and D. Griswold. Strongtalk: typechecking Smalltalk in a production environment. In *OOPSLA’93*, pages 215–230, 1993.
16. Gilad Bracha, Martin Odersky, David Stoutamire, and Philip Wadler. Making the future safe for the past: Adding genericity to the Java programming language. In Craig Chambers, editor, *ACM Symposium on Object-Oriented Programming: Systems, Languages and Applications (OOPSLA)*, volume 33, pages 183–200, Vancouver, BC, October 1998. ACM, ACM SIGPLAN.

17. K. Bruce, A. Schuett, R. van Gent, and A. Fiech. PolyTOIL: A type-safe polymorphic object-oriented language. *ACM Transactions on Programming Languages and Systems*, 25(2):225–290, 2003.
18. Kim Bruce, Luca Cardelli, Giuseppe Castagna, The Hopkins Objects Group, Gary Leavens, and Benjamin C. Pierce. On binary methods. *Theory and Practice of Object Systems*, 1994.
19. Kim B. Bruce. *Foundations of Object-Oriented Languages: Types and Semantics*. MIT Press, 2002.
20. Peter S. Canning, William R. Cook, Walter L. Hill, J. Mitchell, and W. Olthoff. F-bounded polymorphism for object-oriented programming. In *Proc. of Conf. on Functional Programming Languages and Computer Architecture*, 1989.
21. Luca Cardelli. A semantics of multiple inheritance. In *Proc. of the internat. symp. on semantics of data types*, volume 173, pages 51–67. Springer-Verlag, 1984.
22. Luca Cardelli. A semantics of multiple inheritance. *Inform. and Comput.*, 76:138–164, 1988.
23. Luca Cardelli. Structural subtyping and the notion of power type. In *ACM Proceedings of POPL*, 1988.
24. Luca Cardelli, James Donahue, Lucille Glassman, Mick Jordan, Bill Kalsow, and Greg Nelson. *Modula-3 Report (Revised)*, volume 52. Digital Systems Research Center, 1989.
25. Robert Cartwright and Moez A. AbdelGawad. Inheritance *Is* subtyping (extended abstract). In *The 25th Nordic Workshop on Programming Theory (NWPT)*, Tallinn, Estonia, 2013.
26. C. Chambers. Object-oriented multi-methods in Cecil. In *ECOOP*, 1992.
27. C. Clifton, T. Millstein, G. Leavens, and C. Chambers. MultiJava: Design rationale, compiler implementation and applications. *ACM Transactions on Programming Languages and Systems*, 28(3):517–575, 2006.
28. William R. Cook. *A Denotational Semantics of Inheritance*. PhD thesis, Brown Univ., 1989.
29. William R. Cook. On understanding data abstraction, revisited. volume 44, pages 557–572. ACM, 2009.
30. William R. Cook, Walter L. Hill, and Peter S. Canning. Inheritance is not subtyping. In *POPL’90 Proceedings*, 1990.
31. William R. Cook and Jens Palsberg. A denotational semantics of inheritance and its correctness. In *ACM Symposium on Object-Oriented Programming, Systems, Languages and Applications (OOPSLA)*, pages 433–444, 1989.
32. Sophia Drossopoulou, Susan Eisenbach, and Sarfraz Khurshid. Is the java type system sound? *TAPOS*, 5(1):3–24, 1999.
33. Robert Bruce Findler, Matthew Flatt, and Matthias Felleisen. Semantic casts: Contracts and structural subtyping in a nominal world. In *ECOOP 2004–Object-Oriented Programming*, pages 365–389. Springer, 2004.
34. K. Fisher and J. Reppy. The design of a class mechanism for Moby. In *PLDI*, 1999.
35. Matthew Flatt, Shriram Krishnamurthi, and Matthias Felleisen. Classes and mixins. In *Proceedings of the 25th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, pages 171–183. ACM, 1998.
36. Matthew Flatt, Shriram Krishnamurthi, and Matthias Felleisen. A programmer’s reduction semantics for classes and mixins. In *Formal syntax and semantics of Java*, pages 241–269. Springer, 1999.
37. J. Gil and I. Maman. Whiteoak: Introducing structural subtyping in Java. In *OOPSLA*, 2008.

38. James Gosling, Bill Joy, Guy Steele, Gilad Bracha, and Alex Buckley. *The Java Language Specification*. Addison-Wesley, 2014.
39. Ben Greenman, Fabian Muehlboeck, and Ross Tate. Getting f-bounded polymorphism into shape. In *Proceedings of the 35th ACM SIGPLAN Conference on Programming Language Design and Implementation*, PLDI'14, 2014.
40. Atsushi Igarashi, Benjamin C. Pierce, and Philip Wadler. Featherweight Java: A minimal core calculus for Java and GJ. *ACM Transactions on Programming Languages and Systems*, 23(3):396–450, May 2001.
41. X. Leroy, D. Doligez, J. Garrigue, D. Rémy, and J. Vouillon. The Objective Caml system. Available at <http://caml.inria.fr/>.
42. David B. MacQueen. Should ML be object-oriented? *Formal Aspects of Computing*, 13:214–232, 2002.
43. David B. MacQueen, Gordon D. Plotkin, and R. Sethi. An ideal model for recursive polymorphic types. *Information and Control*, 71:95–130, 1986.
44. Boris Magnusson. Code reuse considered harmful, 1991.
45. Donna Malayeri and Jonathan Aldrich. Integrating nominal and structural subtyping. In *ECOOP 2008–Object-Oriented Programming*, pages 260–284. Springer, 2008.
46. Donna Malayeri and Jonathan Aldrich. Is structural subtyping useful? an empirical study. In *ESOP*, 2009.
47. Erik Meijer and Peter Drayton. Static typing where possible, dynamic typing when needed: The end of the cold war between programming languages. In *OOPSLA*, 2004.
48. R. Milner, M. Tofte, R. Harper, and D. MacQueen. *The Definition of Standard ML (Revised)*. MIT Press, 1997.
49. Tobias Nipkow and David Von Oheimb. Java_{light} is type-safe—definitely. In *Proceedings of the 25th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, pages 161–170. ACM, 1998.
50. Martin Odersky. The scala language specification, v. 2.9. <http://www.scala-lang.org>, 2014.
51. Klaus Ostermann. Nominal and structural subtyping in component-based programming. *Journal of Object Technology*, 7(1):121–145, 2008.
52. Benjamin C. Pierce. *Types and Programming Languages*. MIT Press, 2002.
53. Benjamin C. Pierce. Types and programming languages: The next generation. *LICS'03*, 2003.
54. Harry H Porter III. Separating the subtype hierarchy from the inheritance of implementation. *Journal of Object-Oriented Programming*, 4(6):20–29, 1992.
55. Bernhard Reus. Class-based versus object-based: A denotational comparison. *Algebraic Methodology And Software Technology, Lecture Notes in Computer Science*, 2422:473–488, 2002.
56. Bernhard Reus. Modular semantics and logics of classes. In *Computer Science Logic*, volume 2803, pages 456–469. Springer, 2003.
57. Bernhard Reus and Thomas Streicher. Semantics and logics of objects. *Proceedings of the 17th Symp. on Logic in Computer Science (LICS 2002)*, pages 113–122, 2002.
58. Anthony J. H. Simons. The theory of classification, part 1: Perspectives on type compatibility. *Journal of Object Technology*, 1(1):55–61, May-June 2002.
59. Kresten Krab Thorup and Mads Torgersen. Unifying genericity. In *ECOOP 99–Object-Oriented Programming*, pages 186–204. Springer, 1999.